

## MOND PREDICTION FOR THE VELOCITY DISPERSION OF THE ‘FEEBLE GIANT’ CRATER II

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## ABSTRACT

Crater II is an unusual object among the dwarf satellite galaxies of the Local Group in that it has a very large size for its small luminosity. This provides a strong test of MOND, as Crater II should be in the deep MOND regime ( $g_{in} \approx 34 \text{ km}^2 \text{ s}^{-2} \text{ kpc}^{-1} \ll a_0 = 3700 \text{ km}^2 \text{ s}^{-2} \text{ kpc}^{-1}$ ). Despite its great distance ( $\approx 120 \text{ kpc}$ ) from the Milky Way, the external field of the host ( $g_{ex} \approx 282 \text{ km}^2 \text{ s}^{-2} \text{ kpc}^{-1}$ ) comfortably exceeds the internal field. Consequently, Crater II should be subject to the external field effect, a feature unique to MOND. This leads to the prediction of a very low velocity dispersion:  $\sigma_{efe} = 2.1^{+0.9}_{-0.6} \text{ km s}^{-1}$ .

*Keywords:* dark matter — galaxies: dwarf — galaxies: kinematics and dynamics

## 1. INTRODUCTION

The Modified Newtonian Dynamics (MOND) (Milgrom 1983) hypothesizes a modification of dynamics as an alternative to non-baryonic dark matter. While not a complete, relativistic theory intended as a replacement for the standard model of cosmology (Milgrom 2010; McGaugh 2015), MOND makes very specific predictions for the dynamics of low acceleration systems (Milgrom 2014b). A remarkable number of these predictions have been realized (Sanders & McGaugh 2002; Famaey & McGaugh 2012). The predictive ability of MOND is unexpected in the conventional cold dark matter paradigm which, at present, provides no satisfactory explanation for it (Sanders 2009).

In the absence of dark matter, low surface brightness galaxies are necessarily in the low acceleration regime as a direct consequence of the diffuse nature of the luminous mass distribution. Indeed, one of the original predictions of MOND was that low surface brightness galaxies should exhibit large mass discrepancies (Milgrom 1983). Consequently, strong tests are provided by both late type, rotating LSB galaxies (McGaugh & de Blok 1998; Swaters et al. 2010) and diffuse pressure supported systems like dwarf spheroidals (Gerhard & Spergel 1992; Milgrom 1995; Angus 2008; Serra et al. 2009). In the context of MOND, these systems appear to be dark matter dominated<sup>1</sup> because they are in the deep MOND regime,  $g_{in} \ll a_0$ . Numerically,  $a_0 = 1.2 \times 10^{-10} \text{ m s}^{-2} = 3700 \text{ km}^2 \text{ s}^{-2} \text{ kpc}^{-1}$  (Begeman et al. 1991; McGaugh 2011).

At this juncture, it is clear that the dynamical surface density scales with surface brightness (Lelli et al. 2016). This was not anticipated by conventional galaxy formation simulations (Oman et al. 2015), but is expected in MOND (Milgrom 2016). While the success of MOND in fitting rotation curves is widely known (Sanders & McGaugh 2002), it has also had considerable success in predicting the velocity dispersions of the dwarf satellites of Andromeda (McGaugh & Milgrom 2013a,b; Pawlowski & McGaugh 2014). Many of these predictions were made in advance<sup>2</sup> of the measurement of the velocity dispersion, so constitute true *a priori* predictions.

Each and every galaxy poses a distinct test. New objects with extreme properties provide a good opportunity to test the theory by pressing it into a regime where it is not already well tested. The case of the ‘feeble giant’ Crater II (Torrealba et al. 2016) provides such an opportunity.

Crater II is a remarkable object, having a huge linear size (half light radius  $r_h \approx 1.1 \text{ kpc}$ ) for its tiny luminosity ( $M_V \approx -8$ ) (Torrealba et al. 2016). This is like a single globular cluster stretched out to be the size of a galaxy. The only comparable object currently known is And XIX (Collins et al. 2014).

Given its extremely low surface brightness, Crater II should be deep in the MOND regime ( $g_{in} \ll a_0$ ). It is so feeble that despite its great distance from the Milky Way, it may be subject to the External Field Effect

<sup>1</sup> This also applies to Dragonfly 44 (van Dokkum et al. 2016) and any other diffuse extragalactic system.

<sup>2</sup> Ten cases were predicted in advance by McGaugh & Milgrom (2013a), three by McGaugh & Milgrom (2013b), and three more by Pawlowski & McGaugh (2014). See also Pawlowski et al. (2015) for further (as yet mostly untested) predictions.

(EFE). The EFE is a strange feature of MOND in which the dynamics of a dwarf satellite can be affected by the field of its host. Whether the EFE affects Crater II is an important test, as the EFE is unique to MOND and cannot plausibly be attributed to baryonic effects in  $\Lambda$ CDM.

## 2. DATA AND CONTEXT

We adopt for Crater II the values published by [Torrealba et al. \(2016\)](#): a half-light radius  $r_h = 1066 \pm 84$  pc and an absolute magnitude  $M_V = -8.2 \pm 0.1$ . The latter corresponds to a luminosity  $L_V = 1.6 \times 10^5 L_\odot$ . Translating the heliocentric distance to a Galactic coordinate frame, Crater II is about 120 kpc from the Galactic center.

Given its large size, the stars have the opportunity to probe a large range of the gravitational potential well. If Crater II resides in a dark matter sub-halo, one would thus expect a high velocity dispersion for this type of galaxy. The empirical scaling relation between size and velocity dispersion ([Walker et al. 2010](#)) anticipates  $\sigma \approx 17.5 \text{ km s}^{-1}$ . There is large scatter about this relation, so a wide range of velocity dispersions might seem plausible in  $\Lambda$ CDM, which makes no specific prediction for each individual dwarf. Nevertheless, the large size of Crater II anticipates a relatively large velocity dispersion if it resides in a dark matter halo with an NFW-like potential.

Indeed, it came as a surprise that comparably large dwarfs could have rather low velocity dispersions ([Collins et al. 2014](#)). In particular, Crater II is similar to And XIX in having a large size for its luminosity. And XIX and several other dwarfs of Andromeda have low velocity dispersions that were correctly predicted by MOND ([McGaugh & Milgrom 2013a,b](#)). Consequently, it is interesting to make the prediction specifically for Crater II.

## 3. MOND PREDICTION

We predict the velocity dispersion of Crater II using the same methods applied previously to other dwarf satellite galaxies ([McGaugh & Milgrom 2013a](#)). We assume isotropic orbits in a spherical system, and estimate the 3D half-mass radius as  $r_{1/2} = 4r_h/3$  ([Wolf et al. 2010](#)). We adopt a stellar mass-to-light ratio of  $\Upsilon_* = 2_{-1}^{+2} M_\odot/L_\odot$ . The uncertainty in this mass-to-light ratio dominates the formal sources of observational error, which we will ignore bearing in mind the considerable uncertainty in these uncertainties.

The velocity dispersion of an isolated spherical galaxy of mass  $M_*$  in the deep MOND limit is ([Milgrom 1995](#); [McGaugh & Milgrom 2013a](#))

$$\sigma_{iso} \approx \left( \frac{4}{81} a_0 G M_* \right)^{1/4}. \quad (1)$$

Numerically, this becomes  $(\sigma_{iso}/\text{km s}^{-1}) = (M_*/1264 M_\odot)^{1/4}$ . For Crater II,  $M_* \approx 3.3 \times 10^5 M_\odot$ , so  $\sigma_{iso} \approx 4 \text{ km s}^{-1}$ .

This value for the velocity dispersion of  $4 \text{ km s}^{-1}$  is what MOND would predict for a diffuse, isolated galaxy of this stellar mass. This is rather less than one would naturally anticipate for a typical sub-halo ([Boylan-Kolchin et al. 2012](#); [Garrison-Kimmel et al. 2014](#)), and certainly less than expected from the scaling relation of [Walker et al. \(2010\)](#). However, MOND predicts the velocity dispersion to be smaller still if this dwarf is subject to the external field effect (EFE).

The EFE ([Bekenstein & Milgrom 1984](#); [Milgrom 2014a](#)) is a unique feature of MOND. Whether it is present or absent thereby provides a unique test above and beyond tests in isolated systems (e.g., rotation curves). The EFE is a consequence of the nonlinearity<sup>3</sup> of MOND: the acceleration of a star is not simply the vector sum  $\sum_i \vec{g}_{*,i}$  of all other stars acting separately on it as it is in linear theories like Newtonian gravity. It arises when a very low acceleration system (like a dwarf satellite with internal field  $g_{in}$ ) is embedded in the field of a merely low acceleration system (like a host galaxy with external field  $g_{ex}$ ) such that  $g_{in} < g_{ex} < a_0$  ([Milgrom 1995](#)). The imposition of an external field alters the dynamics of a feeble system because it is not as deep in the MOND regime as it would be if isolated.

In the EFE regime, the velocity dispersion estimator of MOND becomes ([McGaugh & Milgrom 2013a](#))

$$\sigma_{efe} \approx \left( \frac{a_0 G M_*}{3 g_{ex} r_{1/2}} \right)^{1/2}. \quad (2)$$

This has the same structure as the Newtonian estimator but with an effective value of Newton's constant enhanced by the MOND interpolation function  $G_{eff} = G/\mu(g/a_0)$ . In the case of Crater II,  $\mu(g/a_0) \approx g_{ex}/a_0$  is an excellent approximation.

For an isolated system, the MOND-predicted velocity dispersion depends only on mass (eq. 1). In the EFE regime, it additionally depends on the size of the system and the external field. Consequently, two dwarfs that are photometrically identical should<sup>4</sup> have different velocity dispersions if one is isolated while the other is subject to the EFE. [McGaugh & Milgrom \(2013b\)](#) identify several such pairs of dwarfs around Andromeda where this predicted difference appears to be reflected in the data.

The exceptional size of Crater II makes it likely to

<sup>3</sup> MOND violates the strong equivalence principle ([Will 2014](#)), but not the universality of free fall ([Milgrom 2015](#)).

<sup>4</sup> This should not happen in linear theories like Newtonian gravity, with or without dark matter.

be subject to the EFE despite its great distance from its host. We estimate the internal acceleration at the half-light radius (McGaugh & Milgrom 2013a) as

$$g_{in} \approx \frac{3\sigma_{iso}^2}{r_{1/2}} \quad (3)$$

where  $\sigma_{iso}$  is calculated with eq. 1. For Crater II,  $g_{in} \approx 34 \text{ km}^2 \text{ s}^{-2} \text{ kpc}^{-1} = 0.009a_0$ . This is lower than all of the many dwarfs considered by McGaugh & Milgrom (2013a) except for And XIX, to which it is equal.

The external acceleration we estimate as

$$g_{ex} \approx \frac{V_{MW}^2}{D_{GC}}, \quad (4)$$

where  $D_{GC} = 120 \text{ kpc}$  is the Galactocentric distance and the circular velocity of the Milky Way at this distance is taken to be  $V_{MW} = 184 \text{ km s}^{-1}$  (McGaugh 2016). This works out to  $g_{ex} \approx 282 \text{ km}^2 \text{ s}^{-2} \text{ kpc}^{-1} = 0.076a_0$ , an order of magnitude larger than the internal acceleration of Crater II. Consequently, Crater II is in the EFE regime.

The predicted velocity dispersion of Crater II is thus

$$\sigma_{efe} = 2.1_{-0.6}^{+0.9} \text{ km s}^{-1}. \quad (5)$$

The uncertainty here represents a factor of two range around the assumed mass-to-light ratio. Observational errors are neglected.

This velocity dispersion is smaller than the isolated MOND case by a factor of two, and nearly an order of magnitude smaller than anticipated by scaling relations (Walker et al. 2009, 2010). MOND predicts that Crater II should be discrepant from these scaling relations in the same sense as And XIX (Collins et al. 2014). The low velocity dispersion of And XIX was correctly predicted by MOND (McGaugh & Milgrom 2013a,b) for the same reason: the external field dominates.

## 4. DISCUSSION

### 4.1. Potential Complications

Our predictions are only as good as the input data and the assumptions that underly them. We assume spherical symmetry, isotropic orbits, and dynamical equilibrium. Deviations from these ideals could affect the predicted velocity dispersion.

At the level of a few  $\text{km s}^{-1}$ , many potential systematics (like interlopers) can affect the measured velocity dispersion. Essentially all act to artificially inflate  $\sigma$  (see discussion in McGaugh & Wolf 2010). Indeed, the motions of binary stars can by themselves contribute at the  $\sim 2 \text{ km s}^{-1}$  level (e.g., McConnachie & Côté 2010; Simon et al. 2011; Walker et al. 2015).

Despite its great distance from the Galactic center, Crater II is so diffuse that it may be subject to tidal effects. Its tidal radius in MOND (Zhao & Tian 2006) is about  $\sim 1.6 \text{ kpc}$ , not vastly larger than its half-light

radius. This is an uncertain calculation specific to its current orbital distance, so must be interpreted with caution. However, should evidence of tidal disruption emerge, it would be more natural in MOND than if Crater II inhabits a dark matter sub-halo, as the latter acts to shield stars from tides.

As a dwarf orbits its host, the external field varies, depending on the eccentricity and orientation of the orbit. Brada & Milgrom (2000) introduced a parameter  $\gamma$  as a measure of the severity of temporal variation in the external field (see their equation 7). It can be interpreted as the typical number of internal orbits a star makes in the time it takes the satellite galaxy to complete one orbit around the host. For large  $\gamma$ , stars complete many internal orbits for every orbit of the dwarf around the host, so the dwarf can adapt to changes in the external field adiabatically. For small  $\gamma$ , dwarfs may not have time to adjust.

Many ultrafaint dwarfs show hints of tidal effects at low values of  $\gamma \lesssim 8$  (McGaugh & Wolf 2010). This suggests a violation of the assumption of dynamical equilibrium. For Crater II,  $\gamma \approx 2$  for a star at the half-light radius, so the assumption of equilibrium is a concern: such a star orbits the Milky Way half as often as it orbits within the dwarf.

There is reason to hope that Crater II may nevertheless provide a robust test. Unlike the ultrafaint dwarfs discussed by McGaugh & Wolf (2010), it is sufficiently far out that the time scales are long. The orbital period at its current distance from the Milky Way is  $\sim 4 \text{ Gyr}$ . Thus it has likely made only a few orbits in a Hubble time, and any non-equilibrium effects may as yet be subtle.

### 4.2. Nominal Expectation in $\Lambda\text{CDM}$

Unlike MOND,  $\Lambda\text{CDM}$  does not make a specific prediction for the velocity dispersions of individual dwarfs. As a population, one expects them to fall along the extrapolation of the stellar mass–halo mass relation (e.g., Moster et al. 2010; Behroozi et al. 2013). Extrapolating from abundance matching to these low mass scales is very uncertain. To complicate matters further, there must be a great deal of scatter in mapping from luminosity to halo mass simply from halo to halo scatter (Tollerud et al. 2011). This can be seen in the simulations of Brooks & Zolotov (2014), in which the characteristic velocity varies by a factor of  $\sim 3$  for galaxies of similar luminosity (see their Fig. 8, which depicts circular speeds at the scale of  $1 \text{ kpc}$  appropriate to Crater II). Consequently, there is no agreed method by which the observed distribution of the stars can be used to predict the velocity dispersions of dwarfs in  $\Lambda\text{CDM}$ . Given the expected scatter, it is not obvious that such an exercise should even be possible, despite the success of

McGaugh & Milgrom (2013a) in doing it.

To get a sense of the expected scale of the velocity dispersion, we first consider pure NFW halos (Navarro et al. 1997) of the appropriate mass scale. Because of the highly non-linear relation between luminosity and mass required by abundance matching (Moster et al. 2010; Behroozi et al. 2013), low luminosity dwarfs are currently expected to reside in halo masses of  $\lesssim 10^{10} M_{\odot}$  (e.g., Brook & Di Cintio 2015; Dutton et al. 2016; Read et al. 2016). At the half light radius of Crater II, an NFW halo with  $M_{200} = 10^{10} M_{\odot}$  has a circular velocity of  $29 \text{ km s}^{-1}$ , corresponding to  $\sigma \approx 17 \text{ km s}^{-1}$ . This corresponds well to the size-velocity dispersion scaling relation of Walker et al. (2010).

Fig. 1 of Boylan-Kolchin et al. (2012) provides a good illustration of how small velocities measured at large radii are problematic for  $\Lambda$ CDM. In order to obtain a velocity dispersion as low as that predicted by MOND, we need to consider smaller NFW halos. Knowing the circular velocity curve of an NFW halo, this is trivial to compute for a specified cosmology (Dutton & Macciò 2014). At  $M_{200} = 10^9 M_{\odot}$ ,  $\sigma \approx 10 \text{ km s}^{-1}$  at  $r \approx 1 \text{ kpc}$ . At  $M_{200} = 10^8 M_{\odot}$ ,  $\sigma \approx 6 \text{ km s}^{-1}$ . We do not reach comparably low velocity dispersions until the absurdly small halo mass  $M_{200} = 10^7 M_{\odot}$  for which  $\sigma \approx 3 \text{ km s}^{-1}$ . At this scale, the rotation curve peaks and begins to decline well within the half-light radius, and the extent of the halo is not great ( $R_{200} \approx 4.5 \text{ kpc}$ ).

So far we have only considered ‘raw’ NFW halos. There is a rich field of work on how such halos may be modified by baryonic effects (e.g., Brooks & Zolotov 2014; Chan et al. 2015; Brook & Di Cintio 2015; Dutton et al. 2016; Read et al. 2016). It is far beyond the scope of this letter to review this topic, which has many different (and often divergent) implementations of feedback and other sub-grid physics. For our purposes, it suffices to note that it may be possible to ascribe a factor of  $\sim 2$  reduction in velocity dispersion to baryonic effects (Read et al. 2016), so perhaps we might anticipate  $\sigma \approx 8 \text{ km s}^{-1}$  rather than  $17 \text{ km s}^{-1}$ . Either way, the expectation of  $\Lambda$ CDM is for a higher velocity dispersion than that predicted by MOND.

#### 4.3. Prediction and Accommodation

It would be good to have a test that clearly distinguished between  $\Lambda$ CDM and MOND. Some observations (Sanders & McGaugh 2002; Famaey & McGaugh 2012) prefer a dark matter interpretation (e.g., clus-

ters of galaxies) while others prefer MOND (e.g., rotation curves). Which interpretation seems preferable depends on how we weigh the various lines of evidence (McGaugh 2015). In general, the dark matter paradigm can accommodate a broader range of phenomena, while MOND has had many predictive success that are unexpected in the context of  $\Lambda$ CDM (Sanders & McGaugh 2002; Famaey & McGaugh 2012; McGaugh 2015).

An important aspect to consider in weighing the evidence is the uniqueness of the prediction each theory makes. MOND makes a very specific prediction for rotation curves that is unique to each individual galaxy (Sanders 2009).  $\Lambda$ CDM does not. It therefore seems strange that out of the enormous parameter space available to galaxies constructed of a baryonic disk plus dark matter halo that the result so often looks like MOND (McGaugh et al. 2016). This is a fine-tuning problem (McGaugh 2015).

Objects like Crater II provide an additional uniqueness test. Conventionally, if Crater II has a velocity dispersion of  $\approx 0.6 \text{ km s}^{-1}$  we would say it is a star cluster with no dark matter. If it has a velocity dispersion of  $4 \text{ km s}^{-1}$ , we would say it has some dark matter. If it has a velocity dispersion of  $8 \text{ km s}^{-1}$ , we would say it has lots of dark matter. If  $\sigma \approx 17 \text{ km s}^{-1}$ , we would declare it to be one of the most dark matter dominated galaxies known. There is no uniquely predicted value; we simply infer the amount of dark matter that corresponds to the observed velocity dispersion.  $\Lambda$ CDM can accommodate any result.

In MOND, the prediction is unique. Barring a drastic failure of the input data and necessary assumptions, the velocity dispersion of Crater II must be  $\sigma_{eff} \approx 2 \text{ km s}^{-1}$ . It should not be  $4 \text{ km s}^{-1}$  as it would be if Crater II were isolated. It certainly should not be significantly larger: a high observed velocity dispersion would falsify MOND.

In contrast, a velocity dispersion as low as  $2 \text{ km s}^{-1}$  would be unprecedented in  $\Lambda$ CDM. The sub-halos in which dwarfs of comparable size and luminosity are imagined to reside are expected to have higher velocity dispersions (Boylan-Kolchin et al. 2012), especially for objects extending over kpc scales. The small amplitude and uniqueness of the value predicted by MOND would thus pose a problem for  $\Lambda$ CDM, were it to be observed.

This publication was made possible through the support of the John Templeton Foundation. The opinions expressed here are those of the authors and do not necessarily reflect the views of the John Templeton Foundation.

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